# Circulating Spin Angular Momentum Modes Using a Discretized Metasurface

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***Abstract*—We propose the spin angular momentum (SAM) space operation of a metasurface, which can realize the circulation of electromagnetic waves with different SAMs. Under a certain cut-off condition to achieve mode selection in a coaxial waveguide, a metasurface can realize the circulation of SAM modes by converting inputs with SAMs of -1, 0, 1 to outputs with SAMs of 0, 1, -1 respectively. To prove the concept, a structure was designed by placing an ideal transmitting metasurface inside a coaxial waveguide. Simulation results show the circulation of three considered SAM modes with near perfect efficiency.**

***Keywords—metasurface, mode conversion, mode circulation, spin angular momentum***

## I. Introduction

Metasurfaces are a kind of subwavelength-thick artificial material layer which can break the limitation of natural materials and provide versatile control of electromagnetic (EM) waves[1]. Under generalized laws of reflection and refraction, the phase discontinuity at an interface between two different media governs the wavefront modulation[2]. Therefore, a metasurface can realize EM wave manipulation by providing phase discontinuity distribution using subwavelength artificial elements. The versatility of metasurfaces are demonstrated by a variety of applications, including anomalous wave reflection and refraction[3-5], antenna beamforming[6, 7], polarization control[8], etc.

The effect of a periodic metasurface acting on an incoming plane wave in free space can be characterized by the *k*-space operation[5]. By suitably designing the *k*-space property of a metasurface and making use of the diffraction modes, a *k*-space mode circulation effect can be observed. However, the mode circulation in the *k*-space does not give obvious circulation effects in plane wave reflection or refraction.

In this work, we propose the circulation in SAM space (*L*-space), which is analogous to the *k*-space circulation. Under certain cut-off condition, a particular metasurface can convert the inputs with SAMs of -1, 0, 1 to outputs with SAMs of 0, 1, -1 respectively, showing circulation effect. As a proof of concept, a structure was designed by placing an ideal transmitting metasurface at a cross-section of a coaxial waveguide. The simulation results show the circulation of the three modes with near perfect conversion efficiency.

## II. *K*-Space Circulation

When a periodic metasurface is illuminated by an incident plane wave in free space, the output will consist of an infinite number of harmonics, as shown in Fig. 1. The *n*-th diffraction mode of the output plane wave will have a tangential wave number of $k_{out,n} = k_{in} + n \cdot k_g$, where $k_{in}$ is the tangential wave number of the incoming plane wave, $k_g = 2\pi / \mathrm{D}$ is the wave number of the metasurface with period D. The blue box in Fig. 1 represents the propagation range of $-k_0 \le k_y \le k_0$, where $k_0$ is the free space wave number. The diffraction modes within the propagation range can scatter into the far field while the ones out of it will become evanescent.

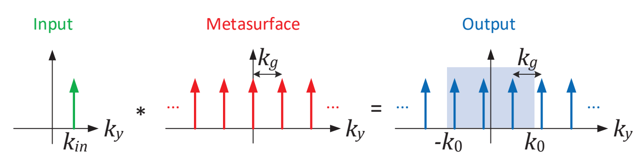


Fig. 1. *k*-space operation of a periodic metasurface that variates along y-direction. Arrows indicate the existence of diffraction modes.

Consider a periodic metasurface with 2*N*+1 diffraction modes falling into the propagation range and only the first-order diffraction mode placed at $k_g$ has non-zero amplitude. Furthermore, the whole *k*-space of the metasurface is periodic of these 2*N*+1 diffraction modes. Then an input with tangential wave number of $mk_g$ will give an output with tangential wave number of $(m+1)k_g$, where *m* is an integer satisfying $-N \le m \le N-1$. Additionally, for an input with tangential wave number of $Nk_g$, the metasurface will give an output with tangential wave number of $-Nk_g$. Therefore, the metasurface realizes circulation of 2*N*+1 modes. $N=1$ gives the simplest case with circulation of only three modes. Fig. 2 shows the *k*-space operation and a schematic diagram of a transmitting metasurface which circulates three modes with tangential wave numbers of $-k_g$ , 0, $k_g$ . As can be seen, a transmitting metasurface with three diffraction modes in the propagation range and featuring *k*-space circulation will deflect incident plane waves with tangential wave numbers of $-k_g$ , 0, $k_g$ to refracted waves with tangential wave numbers of 0, $k_g$ , $-k_g$ respectively.

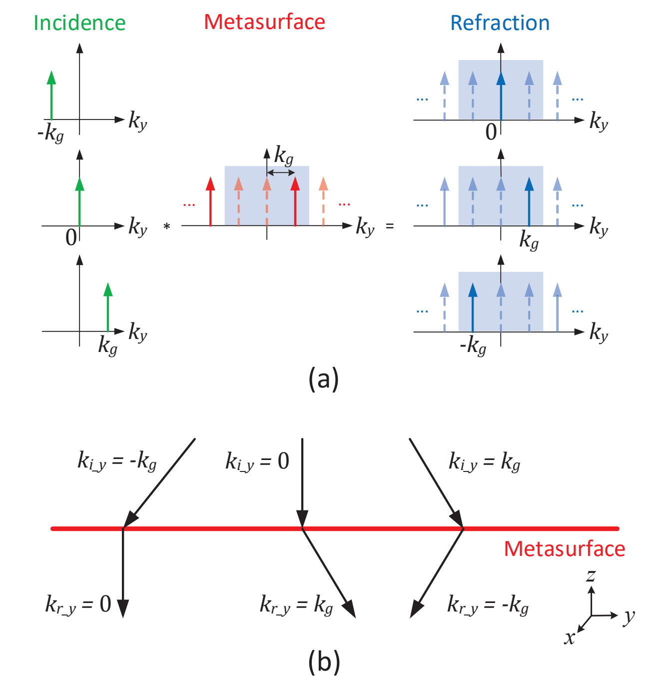


Fig. 2. (a) The *k*-space operation of a metasurface showing the circulation of three diffraction modes. Arrows with solid line represent diffraction modes with non-zero amplitude, while arrows with dashed line represent diffraction modes with zero amplitude. (b) A schematic diagram of a transmitting metasurface upon different incoming plane waves as shown in (a).

Despite the realization of mode circulation in *k*-space, the refraction or reflection of a metasurface upon incidents does not show obvious circulation effect.

## III. L-Space Circulation

Analogous to *k*-space circulation, we propose *L*-space circulation, which is the circulation of EM waves with different SAMs realized by a metasurface. Fig. 3(a) shows the *L*-space operation of a metasurface which circulates three SAM modes, the inputs with SAMs of -1, 0, 1 upon this particular metasurface will give outputs with SAMs of 0, 1, -1 respectively.

As a proof of concept, we designed a structure based on coaxial waveguide with an ideal metasurface. The SAM of a wave propagating in a coaxial waveguide along +z-direction can be expressed as:

$$L = \frac{d\varphi E(\vec{r})}{d\phi} \tag{1}$$

where $\varphi E(\vec{r})$ is the phase of the electric field phasor at place $\vec{r}$ with a subtended angle $\phi$ from the +x axis, as shown in Fig. 3(b). The TEM-mode in a coaxial waveguide has no cut-off and an SAM of 0; the modes with SAMs of -1 and 1 can be synthesized using $H_{11}$-mode. By designing the geometrical parameters of a coaxial waveguide which satisfies the cut-off requirements in Fig 3(a), we can realize the demonstration of SAM mode circulation. Equation (2) gives the approximate cut-off wavelength of $H_{m1}$-mode in a coaxial waveguide with the inner conductor radius and outer conductor radius of $R_1$ and $R_2$ respectively[9]. To realize the cut-off condition of $1 < |L_0| < 2$, we should design the geometrical parameters so that only TEM and $H_{11}$-modes are allowed in the coaxial waveguide, and all the higher modes are cut-off.

$$\lambda_c \cong \pi\left(R_1 + R_2\right)/m \tag{2}$$

Fig. 3(c) shows the structure of our proposed model. SAM mode circulation is achieved using a phase-transmission surface inside a coaxial waveguide. This is achievable rather straightforwardly using a metasurface; in this work we perform the simulations here using an ideal phase shifting boundary to evaluate the effectiveness of SAM mode conversion. The metasurface is ideally realized by three sets of master/slave boundaries providing phase shifts of 0, $\frac{2\pi}{3}$ , $\frac{4\pi}{3}$ respectively, and it has the *L*-space property shown in Fig. 3(a).

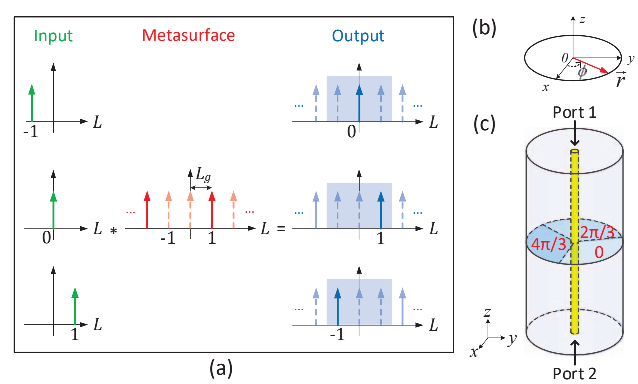


Fig. 3. (a) The *L*-space operation of a metasurface featuring the circulation of three modes with SAMs of -1, 0, 1. (b) SAM of a EM wave propagating along the +z-direction. (c) An ideal model designed to prove the concept of *L*-space mode circulation.

The commercial software Ansys HFSS is used for full-wave electromagnetic simulation. To satisfy the cut-off requirement, the inner conductor radius and outer conductor radius of the coaxial waveguide are designed as 10 mm and 120 mm at the frequency of 1 GHz. The generation mechanism of the three modes with SAMs of -1, 0, 1 is shown in (3), where $m_{01}$ is the TEM-mode in coaxial waveguide, $m_{02}$ and $m_{03}$ are $H_{11}$-modes with a 90-degree phase shift, and $m_1$ , $m_2$ , $m_3$ are the modes with SAMs of -1, 0, 1 respectively. The mode with SAM of 0 is the dominate TEM-mode in the coaxial waveguide, and the modes with SAM of $\mp 1$ are synthesized by the two $H_{11}$-modes with phase differences of $\pm\frac{\pi}{2}$.

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} & j\frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -j\frac{\sqrt{2}}{2} \end{pmatrix} \times \begin{pmatrix} m_{01} \\ m_{02} \\ m_{03} \end{pmatrix} \tag{3}$$

The S-matrix representing the mode conversion efficiency among modes with different SAMs in our proposed model is:

$$S = \begin{pmatrix} 0.0406 & 0.0118 & 0.9971 \\ 0.9990 & 0.0274 & 0.0387 \\ 0.0204 & 0.9995 & 0.0655 \end{pmatrix} \tag{4}$$

where $S_{ij}\left(i, j = 1,2,3\right)$ is the normalized transmission coefficient of mode $j$ input from port 1 to mode $i$ received at port 2. As can be seen, $S_{21}$, $S_{32}$ and $S_{13}$ are close to unity while all the others are very small. Therefore, the proposed model can provide circulation of modes with SAMs of -1, 0, 1 with near perfect efficiency.

## IV. Conclusion

In conclusion, we have demonstrated a simple coaxial waveguide device which performs circular conversion among three waveguide modes with different SAMs. This conversion was achieved using the concept of the discretized metasurface. By properly designing the transmission phase shift, discretization level and waveguide dimensions, we have converted input waves with SAMs of -1, 0, 1 to outputs waves with SAMs of 0, 1, -1 respectively. That a simple, passive and reciprocal surface can perform the circular conversion of SAM modes is surprising, and a strong demonstration of unintuitive opportunities afforded by appropriately discretizing an electromagnetic surface. In ongoing efforts, we aim to implement the transmission metasurface drawing on canonical elements, and design SAM mode circulators and converters in reflection mode and in free space.

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